Abstract—InSAR phase measurements are relative, ambiguous due to $2\pi$ wrapping, and affected by atmospheric and other noise sources. For many real-world applications unambiguous displacement measurements are necessary. Currently available methods for extracting time series of surface displacement, start with unwrapping in time or space domain. We present a new extended Kalman filtering approach to phase unwrapping, which resolves the $2\pi$ ambiguity in a computationally efficient way. Also this algorithm can easily be expanded to use all the data available in space and time dimensions. Several modifications are made to the standard EKF for phase unwrapping. Hence we call our new algorithm as optimized Kalman (OK) filter.

I. INTRODUCTION

SAR images constitute of amplitude and phase information. Amplitude is related to the target’s response to the radar wave, and the phase is relative to the distance between the satellite and the target. SAR interferometry can measure surface deformation looking at the phase change between acquisitions. InSAR phase measurements have been successfully used for applications such as earthquakes, volcano and man-made subsidence or uplift monitoring, and imaging ice flow [1]–[4]. Since the phase information can only be retrieved modulo $2\pi$, it is essential to unwrap these measurements. In the past two decades several methods have been proposed for unwrapping, i.e., branch-cut, minimum cost flow, Kalman filtering, and etc. [5]–[8]. In this paper we describe a new unwrapping method based on the Extended Kalman Filter (EKF). We provide results for one and two dimensional data, i.e. the x-y spatial coordinates, and discuss its application to 3D (2D plus time) unwrapping.

Information on Kalman filters is available on many published sources [9]–[12]. Here we provide only the essential parts of EKF unwrapping.

II. INSAR UNWRAPPING

SAR systems are designed to be phase coherent, such that same acquisition conditions provides the same target response. However, in repeat pass interferometry there are always small perturbations in acquisition parameters; i.e. orbits, water vapour. Interferometric phase ($\phi$) of a target can be described as [13]

$$\phi = \phi_{\text{topo}} + \phi_{\text{defo}} + \phi_{\text{atmo}} + \phi_{\text{orbit}} + \phi_{\text{scat}} + \phi_{\text{noise}}$$  \(1\)

Topographic phase contribution ($\phi_{\text{topo}}$) can be minimized with accurate orbit information and an external digital elevation model (DEM) with high enough resolution. Orbit errors ($\phi_{\text{orbit}}$) generally appear as a long wavelength feature in the interferogram as well as residual topographic fringes. This error can be reduced using more accurate orbit models. Phase contribution due to temporal and spatial changes in the scattering characteristics of the target are noted as $\phi_{\text{scat}}$ and can be considered small for man-made objects. Therefore, for targets with reasonable signal to noise ratio (SNR) it is now only a matter of separating deformation signal ($\phi_{\text{defo}}$) from atmospheric contributions ($\phi_{\text{atmo}}$). Further analysis and implications of (1) will be discussed in the final section of this paper.

III. 1D UNWRAPPING

The simplest case for phase unwrapping is one dimensional, as in the initial unwrapping of the point scatterers in time in the persistent scatterers interferometry (PSI) technique [14], [15]. In order to apply the 1D EKF unwrapping algorithm the phase slope and its noise vectors need to be defined. Phase slope defines how the phase changes as the filter moves to the next state. We calculate phase slope and its noise covariance using the method of Krämer and Loffeld [7], [8].

Fig.1 shows a simple case where a chirp modulated signal is mixed with an additive random phase noise. For example in (a) SNR is 3dB, meaning for a phase change of 1 cm between adjacent measurements the noise strength is 0.707 cm.

When similar analyses are carried out for lower SNR levels, we find that somewhere around SNR=1dB, the EKF result starts to deviate from optimum unwrapping results. Table I shows how the variance changes with EKF for different SNR levels. The results for strong noise case (SNR=0.5 dB) is show in Fig.1(d). The unwrapped result is generally smooth, but shows several phase jumps related to misinterpretation of noisy data.

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Fig. 1. For SNR 3 dB the unwrapped result perfectly matches the input phase (a). For SNR 0.5 dB the unwrapped result is generally smooth, but shows several phase jumps related to misinterpretation of noisy data (b-d).

<table>
<thead>
<tr>
<th>SNR [dB]</th>
<th>$\sigma^2$ Before EKF [rad]</th>
<th>$\sigma^2$ After EKF [rad]</th>
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</thead>
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<tr>
<td>3</td>
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<td>0.2288</td>
</tr>
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<tr>
<td>0.5</td>
<td>3.3458</td>
<td>0.8941</td>
</tr>
</tbody>
</table>

### IV. 2D Unwrapping

#### A. Phase slope estimation

2D unwrapping can be achieved by summing information from both dimensions with proper weights to predict the estimate for the next state [7]. For any given pixel in a 2D grid there are four near neighbors (common edges) and four far neighbors (common corners). We expand the relations in [7] to make use of information from all the neighbors. The weighting factors for all directions are proportional to their distance from the current state.

Since new estimations are only based on the previously estimated value and the phase slope, if the phase estimation for the current state is correct our future predictions can only be wrong due to phase slope. Two dimensional FFTs are required for phase slope estimators using PSD, however; the computational cost of this operation is quite high. It is even more costly to calculate FFTs of datasets with missing data points (incoherent areas). For this purpose, we use a series of simple linear operations to estimate phase gradient as well as its error estimate, based on shifting the phase to slide the fringes, hence called “shift and slide” algorithm.

Wrapped data has phase jumps at $-\pi + \pi$ transitions. In noisy areas it is hard to know the exact location of the phase jump causing errors in gradient calculation. Shift and slide algorithm separates fringes from each other by masking out the phases between +0.5$\pi$ to $-0.5\pi$ (such that we avoid $-\pi + \pi$ transitions). Later we calculate the gradient on the remainder of the signal where the phase jumps do not exist. We then shift the phase of the original signal by adding $\pi$ and calculate the gradient the same way. Shifting the phase essentially moves the phase jumps in the image. First row of Fig.2 shows this effect. The gradients shown in Fig.2(c-d) are then merged allowing us to calculate continuous phase slope free of errors associated with phase jumps.

However cases exist where the fringes are not separated due to limitations on the resolution or strong noise. These spurious calculations can be removed using an adaptive moving filter. We distinguish between noisy and coherent areas by looking at the phase distribution. Noisy areas have a uniform phase distribution whereas coherent areas have a gaussian distribution even in a small window as shown in Fig.3. Benefiting from this feature, we can develop an adaptive moving window filter which rejects areas with uniform distribution, as well as points far away from the mean depending on the standard deviation $\sigma$ of the gaussian distribution. Band width ($BW$) of the filter can be formulated as:

$$BW = \pi - \left(\frac{\sigma}{2\pi/3}\right) \times \pi$$

#### B. Automated Error Correction

Unwrapping errors are inevitable in noisy data so we included an automated error detection and correction algorithm to the optimized Kalman (OK) filter. The OK filter minimizes the error in the wrapped domain, which means, it neglects the errors that do not manifest themselves in the wrapped solution. Therefore, in cases where SNR is low, it might go up-cycle(+$\pi$) instead of down-cycle($-\pi$) or vice-versa. In order to reduce such errors we put a limit on the Kalman gain. Nyquist-Shannon sampling criterion states that at least two measurements are required in one cycle, providing a hard bound of $\pi$ on the phase change between each state [16].

If the calculated phase difference between states are above this threshold, the OK filter will not return any solution implicitly masking incoherent areas. Furthermore, in both wrapped and unwrapped interferograms the local phase slope should be similar. Therefore we re-calculate phase slope of the

Fig. 2. A simple phase shift slides the fringes (top row). Once the fringes are separated, a simple gradient operation can be used to estimate phase slope. It is easy to see that when put on top of each other, gradients will match each other perfectly (bottom row). Synthetic data is 512 by 512 and slope is calculated in horizontal dimension.
unwrapped point with its known neighbors. If the gradients between the wrapped and unwrapped solutions are not similar it indicates a misjudgement in the filter and the solution is skipped until we can approach the same point from another direction.

We further optimize the filter by letting it follow the minimum gradient instead of a row/column-wise direction. This approach minimizes the errors due to under-sampling, and noise. Unwrapping errors are particularly common for pixels which the OK filter only approaches from a single direction. We define this as possible disconnects in the unwrapped solution and separate such areas into classes. If there are any, the phase jumps between these classes are calculated and corrected using a linear inversion.

V. POTENTIAL FOR 3D UNWRAPPING

Most unwrapping algorithms (e.g. branch-cut, minimum cost flow [5], [6]) work on 2D single interferograms leading to errors in phase closure when multiple interferograms are considered [17]. These errors also occur in interferograms of fast deforming areas [18]. By including information from the third (temporal) dimension during unwrapping, these errors can be reduced. As discussed earlier (1) not much is known about the topographic and the atmospheric phase contributions. However, since they have different characteristics in space and time, they can be best estimated using an approach that combines data from all dimensions. For space-time unwrapping we start with single master stacks used in PSI technique [14], [15] which has a temporal sequence ideally suited to progressive characteristics of OK filter. The interferograms are ordered in time and they all share the master scene’s spatial axes. This is the current focus of research, and preliminary results will be presented in the conference.

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REFERENCES


